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The views expressed in this paper are those of the authors.
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Abstract

The authors examine the impact of multinational enterprises (MNEs) on exchange rate pass-through in an environment where an MNE engages in Cournot (quantity) competition with domestic and foreign rivals. The MNE differs from its competitors because it has a lower marginal cost as a result of increased efficiency, and economies of scope as a result of operating in two markets. An MNE can also choose to locate its production for the foreign market domestically (in the location of the MNE's parent), or in the foreign country (the location of the subsidiary). When it locates all its production domestically, it engages in *intrafirm trade* (IT) in final goods. Otherwise, it is said to engage in *international production* (IP). Consistent with other studies on exchange rate pass-through under imperfect competition, the authors' analysis shows that exchange rate pass-through into domestic and foreign prices is incomplete. Moreover, the presence of an MNE increases the sensitivity of domestic market prices, and reduces the sensitivity of foreign market prices, to exchange rate movements, relative to arm's-length trade. Furthermore, IT domestic and foreign prices are more sensitive to exchange rate movements than their IP counterparts, and react in the opposite direction. The authors' results indicate that it is important to distinguish between the domestic and the foreign market when looking at the sensitivity of prices and their direction of change. This could potentially explain why some empirical studies find IT prices more sensitive to exchange rate movements and others find them less sensitive.

JEL classification: F23, L16
Bank classification: Economic models; Exchange rates; Market structure and pricing

Résumé

Les auteures étudient les effets de l'existence de multinationales sur le degré de transmission des mouvements de change aux prix dans un modèle de concurrence par les quantités (ou concurrence à la Cournot) où une multinationale affronte des rivales à la fois sur son marché intérieur et sur un marché extérieur. Cette multinationale se distingue de ses concurrentes par une plus grande efficience, qui lui vaut un coût marginal plus bas, et par des économies de gamme, qu'elle doit à sa présence sur deux marchés. Outre ces atouts, la multinationale peut décider de centraliser sa production pour le marché extérieur dans le pays de domiciliation de sa société-mère ou de la délocaliser à l'étranger, sur le territoire de sa filiale. Quand toute l'activité de production s'effectue sur le territoire de la société-mère, on dit qu'il y a *commerce intra-entreprises* de biens finaux; dans le cas contraire, on parlera de *production internationale*. Comme chez d'autres

chercheurs, l'analyse des auteures montre que les variations de change ne se répercutent pas intégralement sur les prix intérieurs et extérieurs en contexte de concurrence imparfaite. Qui plus est, comparativement à des conditions de pleine concurrence, l'existence d'une multinationale rend les prix intérieurs plus sensibles aux fluctuations du taux de change, et les prix à l'étranger moins sensibles. De surcroît, les prix sur les deux marchés réagissent davantage aux mouvements de change dans le cas du commerce intra-entreprises que dans celui de la production internationale, mais ils évoluent en sens opposé. Les auteures en concluent qu'il est important pour quiconque étudie la sensibilité des prix et le sens de leurs variations de veiller à différencier les marchés intérieur et extérieur. Cette distinction pourrait expliquer pourquoi les prix observés dans le cas du commerce intra-entreprises sont plus sensibles selon certaines études empiriques et moins selon d'autres.

Classification JEL : F23, L16
Classification de la Banque : Modèles économiques; Taux de change; Structure de marché et fixation des prix

1. Introduction

Multinational enterprises (MNEs) play an important role in international trade. Recently, MNEs' pricing behaviour has attracted a lot of attention from both policy-makers and researchers. To the extent that MNEs are different from other firms, one would expect that their trade and pricing patterns would also be different. A few empirical studies have attempted to compare the responsiveness of MNEs' trade prices with arm's-length trade (AT) prices, but no theoretical analyses of this exist.[1] In this paper, we develop a model of an MNE and attempt to shed some light on the issue of the sensitivity of MNEs' trade prices to exchange rate movements. In particular, we examine how the presence of an MNE affects the exchange rate pass-through relative to AT, and how the MNE's location of production matters for exchange rate pass-through.

Exchange rate pass-through into consumer prices has fallen considerably in Canada and other industrialized countries over the past two decades. One potential explanation for this decline is the increasing importance of intrafirm trade (IT).[2] The existing evidence shows that MNEs' exports have increasingly shifted away from AT towards IT (Rangan, 2001). The empirical evidence, otherwise limited, on the sensitivity of IT prices to exchange rate pass-through is somewhat mixed. For example, Clausing (2001) finds that IT prices are more sensitive to exhange rate movements than AT prices. Rangan and Lawrence (1999) also show that, relative to AT, imports (quantities) to the United States by MNEs exhibit both stronger and faster responses to exchange rate changes, which the authors attribute to informational advantages arising from multinational operations. However, Pain (2002) finds the opposite for the United Kingdom: IT prices are less sensitive to exchange rate movements than AT prices.

MNEs can organize their production in a number of different ways. An MNE can set up production at one location and transfer finished goods among its different branches. This is the case of IT. An MNE can also undertake production locally for sale in the local market. We refer to this scenario as international production (IP). According to UNCTAD

[1] In arm's-length trade, an exporter sells a product to a local firm that is responsible for the retailing of that product. The exporter has no access to the local market.

[2] Other possible explanations for the decline of the exchange rate pass-through suggested in the literature are: (i) the transition to a low-inflation environment in industrialized countries; (ii) substitution by consumers to lower-priced items, substitution by retailers to lower-cost suppliers, and greater productivity improvements by retailers during periods of depreciation of the national currency; (iii) the shift in the composition of imports in industrialized countries towards sectors with lower pass-through. A Canadian study by Lapham (2004) shows that both lower inflation and restructuring in the retail sector explain the observed lower pass-through to consumer prices in Canada.

1998-2004 reports. IP has increased substantially in the past decade. Finally, an MNE can establish different stages of production in different locations and this will entail IT in intermediate products. We do not examine this case because we do not model intermediate production in this paper.[3] Instead, we focus on an MNE that produces a final good for two locations, domestic and foreign, using local resources at a constant input price at the point of production. The MNE competes with local producers in both markets, but is able to treat the domestic and foreign markets as completely segmented. We consider two different production scenarios: (i) IT under which production is undertaken at the location of the MNE's parent company (resulting in trade of the final good between the two countries), and (ii) IP in which production takes place locally in the location of sales.

The distinguishing features of our model of MNE are: (i) economies of scope or complementarity in the production of outputs, (ii) competition in both markets, and (iii) differential taxation of MNE profits in the two countries.

The notion of economies of scope is related to intangibles such as research and development (R&D), advertising, marketing, distribution, and management services that an MNE is able to share across plants, avoiding duplication of such expenditures (Markusen, 1984). Desai, Foley, and Hines (2005) provide empirical evidence of a complementary relationship between foreign and domestic investment for U.S. MNEs. Their study implies that MNEs combine home production with foreign production to generate a final product at a lower cost than would be possible with production in just one country.

In our model, the MNE competes in quantity (Cournot competition) with local producers in both markets. This is a departure from the MNE literature in which it is assumed that the MNE is a monopolist in both the domestic and the foreign markets. Following a stream of the industrial organization pricing-to-market literature, we choose to work under the assumption of quantity (Cournot) competition rather than that of price (Bertrand) competition (Lapham, 2004; Dornbusch, 1987; Feenstra, Gagnon, and Knetter, 1996; Bodnar, Dumas, and Marston, 2002).

As is common in the MNE literature, the MNE in our model is subject to differential taxation of profits in the two countries in which it operates. In this context, cost assignment rules (also referred to as transfer pricing rules) become particularly important. The common cost-splitting rule we employ breaks up the cost into two: the *stand-alone cost* the MNE

[3]An increasing proportion of IT by MNEs is accounted for by intermediate inputs, and we hope to examine intermediate production in future research.

incurs in the case where it produces only for the domestic market, and the *incremental cost* the MNE incurs when producing for the foreign market as well. This rule is realistic and has been used in other studies of MNEs, such as Calzolari and Scarpa (2004) and Eden (1998). However, contrary to other MNE papers that focus on the regulation of MNEs by means of taxation, in our paper we assume that profit taxes are exogenous.[4] We incorporate taxes in our analysis to investigate the effects of these on the sensitivity of prices with respect to the exchange rate. There exists substantial evidence of the relationship between countries' tax rates and prices of intrafirm transactions. One such study is Clausing (2003), which shows that as the tax rates of the destination and origin countries are lower, U.S. intrafirm export prices are lower and U.S. intrafirm import prices are higher. It is therefore important to include taxes in any analysis of MNEs' pricing behaviour.

Our paper combines two strands of literature: the MNE literature and the literature on exchange rate pass-through. To our knowledge, our analysis is novel and no other analytical studies of the effect of MNEs' pricing behaviour on exchange rate pass-through exist. The only related paper that we came across is Hegji (2003). Hegji develops a model in which an MNE produces locally and exports some of the output to a foreign subsidiary, which incurs additional costs. This framework allows Hegji to derive simple expressions for the exchange rate pass-through in terms of elasticities of demand and marginal costs. Our paper builds on Hegji's by introducing a rationale for the existence of MNEs (economies of scope), competition in both markets, and considers both IT and IP as alternative means of delivering goods and services.

Consistent with other studies on exchange rate pass-through under imperfect competition, our analysis shows that exchange rate pass-through into domestic and foreign prices is incomplete. Moreover, the presence of an MNE increases the sensitivity of domestic market prices, and reduces the sensitivity of foreign market prices, to exchange rate movements, relative to AT. Furthermore, IT domestic and foreign prices are more sensitive to exchange rate movements than their IP counterparts, and react in the opposite direction.

Our results indicate that it is important to distinguish between the domestic market (the location of the MNE's parent) and the foreign market (the location of the subsidiary) when looking at the sensitivity of prices and their direction of change. This could potentially explain why some empirical studies find IT prices more sensitive to exchange rate movements and others find them less sensitive. While our approach does not allow us to directly link the

[4]The literature on the regulation of MNEs is extensive. See, for example, Calzolari and Scarpa (2004), Calzolari (2004), Bond and Gresik (1996), Dasgupta and Sengupta (1995), and Gresik and Nelson (1994).

observed decline in exchange rate pass-through to the increasing importance of IT, it does allow us to shed some light on how the presence of an MNE and its location of production can affect exchange rate pass-through compared with AT.

This paper is organized as follows. In section 2 we describe the model, and in subsection 2.1 we derive the equilibrium outputs and exchange rate pass-through in both markets. In subsection 2.2, we compare outputs and exchange rate pass-through across cases. In section 3 we offer some conclusions.

2. The Model

In this model, we consider an MNE that sells its products in two locations, domestic (where its parent company is located) and foreign. It supplies outputs y_d to its domestic market and y_f to its foreign market. In each of these markets, it faces Cournot competition from local firms that produce homogeneous products. We assume that there are n identical local firms in the domestic market, each of which supplies the quantity y_{-d}, and m identical local firms in the foreign market, each of which supplies the quantity y_{-f}. We allow the MNE to differ from its local competitors in its production technology, and we assume that the domestic and foreign markets are segmented: that is, demands are independent.

We consider two different cases for the location of production by the MNE: IT and IP. In the IT case, the MNE produces $y_d + y_f$ in its domestic (parent company) location and transfers y_f to its foreign affiliate for sale in the foreign market. In the IP case, the MNE produces y_d domestically and y_f through its foreign affiliate in its foreign location. The MNE's production technology is summarized by a minimum cost function, $C(\varepsilon, y_d, y_f, \beta)$, where $\varepsilon \in \{e, 1\}$ and the exchange rate, e, is the price of the domestic currency divided by the price of foreign currency. For simplicity, we assume the domestic currency as the numeraire and express the MNE's profit and cost functions in terms of the domestic currency. β is an efficiency parameter, which we assume is observable to the MNE and its competitors in both the domestic and foreign markets.

The MNE's cost function satisfies the following properties: $C_{y_i} > 0$, $C_\beta > 0$, $C_{\beta y_i} \geq 0$, $C_{y_i y_i} \leq 0$, and $C_{y_i y_j} = \psi < 0$, a constant, for $i \neq j$. For $\varepsilon = e$, we also assume that $C_e > 0$, $C_{ey_d} = 0$, and $C_{ey_f} > 0$. The assumption $\psi < 0$ reflects the fact that there are economies of scope for the MNE. That is, y_d and y_f are complements in production. Complementarity can arise when the MNE uses common inputs such as R&D, brand name, and reputation.

Finally, where results are clearer with specified functional forms, we assume that the cost function takes the following functional form:

$$C(\varepsilon, y_d, y_f, \beta) = \beta(y_d + \varepsilon y_f) + \upsilon y_d y_f, \tag{1}$$

which satisfies all the required restrictions. $\varepsilon = 1$ corresponds to the case of IT (since this indicates that the cost of producing y_f is incurred in domestic currency) and $\varepsilon = e$ corresponds to the case of IP. The AT case corresponds to $\upsilon = 0$ and $\varepsilon = 1$. This is the case of a purely domestic exporter that produces in its domestic location and exports its output to the foreign market.

The cost assignment rule we introduce breaks up the common cost into two components. The first component is the stand-alone cost (SAC), and it is defined as the cost the MNE incurs if it produces for the domestic market alone, that is, $y_f = 0$:

$$SAC(\varepsilon, y_d, 0, \beta) = \beta y_d. \tag{2}$$

The second component is the incremental cost (IC), and it is the additional cost the MNE incurs if it produces for the foreign market as well. The incremental cost is thus:

$$IC(\varepsilon, y_d, y_f, \beta) = C(\varepsilon, y_d, y_f, \beta) - SAC(\varepsilon, y_d, 0, \beta) = \beta\varepsilon y_f + \upsilon y_d y_f. \tag{3}$$

The incremental cost has two terms. The first term, $\beta\varepsilon y_f$, is positive, and the second term, $\upsilon y_d y_f$, is negative, since $\upsilon < 0$, reflecting the economies-of-scope (or production-complementarity) assumption. In the case of AT, $\upsilon = 0$ and $\varepsilon = 1$, so the foreign cost for a domestic exporter is just $\beta y_f > 0$. The MNE, however, has economies of scope and is able to produce at a lower foreign cost than the domestic exporter.

For simplicity, we assume that the MNE's competitors—domestic and foreign—have constant marginal costs of production equal to 1. We also assume that the MNE is more efficient than its competitors; that is, $\beta \leq 1$. This is a reasonable assumption given the existing empirical evidence that exporters and, a fortiori, MNEs are more efficient than purely domestic firms.[5]

Market conditions in the domestic and foreign markets are given by inverse demand functions. We assume linear demands and allow the two markets to differ along a scale

[5]See, for example, Bernard and Jensen (1995, 1997a,b,c), Richardson and Rindal (1995), Bernard and Wagner (1997), Bernard, Jensen, and Wagner (1997), and Aw and Hwang (1995).

(intercept) and an elasticity (slope) component. The inverse demand function in the domestic market is given by $P_d = a - bY_d$, where $Y_d = y_d + ny_{-d}$ is total output, and n is the number of firms operating in d, each producing output y_{-d}. Similarly, the inverse demand function in f is $P_f = h - kY_f$, where $Y_f = y_f + my_{-f}$ is total output, and m is the number of firms operating in f, each producing output y_{-f}. We assume that $a > 1$ and $h > 1$, which is necessary for the existence of local firms with marginal cost of production equal to 1.

The MNE is subject to profit taxes, t_d and t_f, in the domestic and foreign market, respectively.

The MNE is risk neutral and its objective is to choose quantities, y_d and y_f, to maximize global profits:

$$\begin{aligned}\Pi(\varepsilon, y_d, y_f, \beta) &= (1-t_d)(P_d y_d - SAC) + (1-t_f)(\varepsilon P_f y_f - IC) \\ &= [(1-t_d)P_d y_d + (1-t_f)\varepsilon P_f y_f] - [(1-t_d)SAC + (1-t_f)IC].\end{aligned} \tag{4}$$

The terms in the first square brackets represent the after-tax revenue and the terms in the second square brackets represent the after-tax cost of the MNE.

The MNE's competitors in the domestic market simultaneously choose y_{-d} to maximize profits:

$$(P_d - 1)y_{-d}. \tag{5}$$

and its competitors in the foreign market simultaneously choose y_{-f} to maximize profits:

$$(P_f - 1)y_{-f}. \tag{6}$$

2.1 Equilibrium output and exchange rate pass-through

The MNE simultaneously chooses y_d and y_f to maximize global profits, taking as given the output levels, y_{-d} and y_{-f}, chosen by the domestic and foreign competitors. Assuming interior solutions, we have two first-order conditions associated with the MNE's problem, (7)–(8), and two first-order conditions associated with its competitors in the domestic and

foreign markets, (9)–(10):

$$(1-t_d)(P'_d y_d + P_d) = (1-t_d)SAC'_{y_d} + (1-t_f)IC'_{y_d}, \tag{7}$$

$$eP'_f y_f + eP_f = IC'_{y_f}, \tag{8}$$

$$P'_d y_{-d} + P_d = 1, \tag{9}$$

$$P'_f y_{-f} + P_f = 1. \tag{10}$$

For future reference, it is useful to have a closer look at the MNE's first-order conditions (7) and (8). The left-hand side in (7) is the MNE's after-tax marginal revenue with respect to y_d, and the right-hand side is the MNE's after-tax marginal cost with respect to y_d. Both domestic and foreign taxes affect the MNE's marginal cost of domestic output, since the cost assignment rule that breaks up total cost into the stand-alone cost and incremental cost does not take into account production complementarities, in the sense that foreign output decreases the marginal cost of domestic output. It is this economies-of-scope effect that results in foreign taxes entering into the MNE's first-order condition for domestic output. On the other hand, both the marginal revenue and marginal cost of foreign output are reduced by $(1-t_f)$, and this drops out of the first-order condition for foreign output, which is why taxes do not influence the MNE's equilibrium choice of foreign output directly (it does so only indirectly through changes in domestic output).

The solution to the Cournot games in the domestic and foreign market is obtained by solving equations (7)–(10) simultaneously. The functional forms also allow us to derive the expressions for competitors' outputs, total market outputs, and market prices as functions of the MNE's (or arm's-length exporter's) outputs, which we present in Lemma 1.

Lemma 1 *Equilibrium quantities, prices, and demand elasticities, expressed in terms of the MNE's quantities, y_d and y_f, are given by*

$$y_{-d} = \frac{a-1-by_d}{b(n+1)}, \quad y_{-f} = \frac{h-1-ky_f}{k(m+1)}, \tag{11}$$

$$Y_d = \frac{n(a-1)}{b(n+1)} + \frac{y_d}{n+1}, \quad Y_f = \frac{m(h-1)}{k(m+1)} + \frac{y_f}{m+1}, \tag{12}$$

$$P_d = \frac{n+a-by_d}{n+1}, \quad P_f = \frac{m+h-ky_f}{m+1}. \tag{13}$$

Proof: The proof is relegated to Appendix A. *Q.E.D.*

We denote by $E_{P_i,e} = \partial(ln\ P_i)/\partial(ln\ e)$ the exchange rate pass-through into market i prices, for $i = d, f$. Proposition 1 gives the general expressions for the pass-through elasticities.

Proposition 1 *The exchange rate pass-through into foreign and domestic prices is:*

$$E_{P_i,e} = E_{y_i,e} \cdot \eta_{P_i,y_i}, \quad i \in \{d, f\} \tag{14}$$

where $E_{y_i,e} = -\partial(ln\ y_i)/\partial(ln\ e)$ *is the elasticity of the MNE's output in country* i *with respect to the exchange rate,* $\eta_{P_i,y_i} = -\partial(ln\ P_i)/\partial(ln\ y_i)$ *is the elasticity of market demand* i, $i \in \{d, f\}$ *with respect to the MNE's output in market* i.[6]

Proof: The proof is relegated to Appendix B. *Q.E.D.*

Proposition 1 shows the exchange rate pass-through mechanism by breaking it down into two factors. First, a change in the exchange rate affects the MNE's outputs. Second, the change in outputs affects the elasticity of market demand with respect to the MNE's outputs.

In the pricing-to-market literature, the exchange rate pass-through is typically expressed in terms of markups over marginal costs. We can also express the exchange rate pass-through elasticities as a function of firms' cost elasticities with respect to the exchange rate, and show that exchange rate pass-through is proportional to the elasticity of marginal cost with respect to the exchange rate:

$$E_{P_d,e} = \frac{(1-t_f)/(1-t_d)C'_{y_d}}{a + nC'_{y_{-d}} + (t_f - t_d)/(1-t_d)SAC'_{y_d} + (1-t_f)/(1-t_d)C'_{y_d}} \cdot \frac{\partial\ ln\ C_{y_d}}{\partial\ ln\ e}, \tag{15}$$

$$E_{P_f,e} = \frac{C_{y_f}/e}{b + mC'_{y_{-f}} + (C_{y_f}/e)} \cdot \frac{\partial\ ln\ (C_{y_f}/e)}{\partial\ ln\ e}. \tag{16}$$

[6]The elasticity of the market demand i, $i \in \{d, f\}$ with respect to the MNE's output in market i is positively related to the market demand elasticity:

$$\eta_{P_d,y_d} = \frac{1}{n+1} \cdot \frac{y_d}{Y_d}\eta_d = \frac{by_d}{by_d + n(a-1)}\eta_d,$$

$$\eta_{P_f,y_f} = \frac{1}{m+1} \cdot \frac{y_f}{Y_f}\eta_f = \frac{ky_f}{ky_f + m(b-1)}\eta_f,$$

where $\eta_i = \partial(ln\ P_i)/\partial(ln\ Y_i)$, $i = d, f$.

With some manipulation, we can derive the expressions in Lemma 2.

Lemma 2 *The price elasticities with respect to the MNE's outputs are:*

$$\eta_{P_d,y_d} = \frac{by_d}{a+n-by_d} > 0. \tag{17}$$

$$\eta_{P_f,y_f} = \frac{ky_f}{h+m-ky_f} > 0. \tag{18}$$

Proof: The proof is relegated to Appendix C. *Q.E.D.*

Due to economies of scope in the MNE's cost function ($\upsilon \neq 0$), the price elasticities of the MNE's domestic and foreign output are related. Lemma 3 gives the relationship between the price elasticities of the MNE's domestic and foreign output.

Lemma 3 *The MNE's output elasticities in the domestic and foreign markets are related:*

$$E^j_{y_d,e} = -\upsilon \frac{1-t_f}{1-t_d} \frac{n+1}{b(n+2)} \cdot \frac{y_f}{y_d} E^j_{y_f,e}. \tag{19}$$

$$E^j_{y_f,e} = -\frac{m+1}{ek(m+2)} \left[\upsilon \frac{y_d}{y_f}(1+E^j_{y_d,e}) + \frac{\beta}{y_f} I_\varepsilon \right]. \tag{20}$$

where $j \in \{AT, IT, IP\}$, and I_ε is an indicator function equal to one for $\varepsilon = 1$ and zero for $\varepsilon = e$.

Proof: The proof is relegated to Appendix D. *Q.E.D.*

Lemmas 2 and 3 allow us to predict that the MNE changes the quantities it supplies to both the domestic and foreign market in the same direction in response to an exchange rate shock. Consequently, prices in both markets move in the same direction when an MNE is present. Under AT, however, domestic market prices are invariant to exchange rate movements. These results are summarized in Proposition 2.

Proposition 2 *The following is true:*

(i) $E^j_{y_d,e}$ and $E^j_{y_f,e}$ have the same sign, for $\upsilon < 0$, $j \in \{IT, IP\}$.

(ii) $E^j_{P_d,\varepsilon}$ *and* $E^j_{P_f,\varepsilon}$ *have the same sign, for* $v < 0$, $j \in \{IT, IP\}$.

(iii) $E^{AT}_{P_d,\varepsilon} = 0$ *and* $E^{AT}_{P_f,\varepsilon} < 0$.

Lemma 3 implies that, for $v < 0$, $E^j_{y_d,\varepsilon}$ and $E^j_{y_f,\varepsilon}$, $j \in \{IT, IP\}$ has the same sign. Since elasticities η_{P_i,y_i} are positive by definition, Lemmas 2 and 3 together imply that the exchange rate pass-through into domestic and foreign prices also has the same sign. We can also see from Lemma 3 that, under AT, which corresponds to $v = 0$ and $\varepsilon = 1$, the exporter's output, y_d, is invariant to the exchange rate; that is, $E^{AT}_{y_d,\varepsilon} = 0$, and $E^{AT}_{y_f,\varepsilon} < 0$. This implies that the exchange rate pass-through into domestic and foreign prices is $E^{AT}_{P_d,\varepsilon} = 0$ and $E^{AT}_{P_f,\varepsilon} < 0$, respectively.

Solving equations (7)–(10) for y^j_d and y^j_f, we obtain Proposition 3.

Proposition 3 *Equilibrium quantities in the domestic and foreign markets, respectively, are*

$$y^j_d = \frac{1}{D}\{(1-t_d)ek(m+2)[a+n-\beta(n+1)] - (1-t_f)v(n+1)[e(h+m)-\varepsilon\beta(m+1)]\}, \quad (21)$$

and

$$y^j_f = \frac{1}{D}\{(1-t_d)b(n+2)[e(h+m)-\varepsilon\beta(m+1)] - (1-t_d)v(m+1)[a+n-\beta(n+1)]\}, \quad (22)$$

for $j \in \{AT, IT, IP\}$, *where* $D \equiv (1-t_d)ekb(n+2)(m+2) - (1-t_f)v^2(n+1)(m+1) > 0$.[7]

We can easily show that $y^j_d < (a+n)/2b$ and $y^j_f < (h+m)/2k$, which implies that $\eta_{P_i,y_i} < 1$, for $i \in \{d, f\}$.

For future reference, it is useful to obtain the comparative statics properties of the equilibrium output levels, y^j_d and y^j_f. They are summarized in Lemma 4.

Lemma 4 *The MNE's output,* y^j_i, $i \in \{d, f\}$, $j \in \{AT, IT, IP\}$, *is increasing in* a, b, n, m,

[7] See Appendix E for a proof and expressions for y_{-i}, Y_i, Y_{-i}, P_i, and P_{-i}, for $i \in \{d, f\}$.

and t_d, and decreasing in b, k, β, v, and t_f:

$$\frac{\partial y_i^j}{\partial a} \geq 0. \quad \frac{\partial y_i^j}{\partial h} \geq 0. \quad \frac{\partial y_i^j}{\partial b} \leq 0. \quad \frac{\partial y_i^j}{\partial k} \leq 0. \tag{23}$$

$$\frac{\partial y_i^j}{\partial n} \geq 0. \quad \frac{\partial y_i^j}{\partial m} \geq 0. \quad \frac{\partial y_i^j}{\partial t_d} \geq 0. \quad \frac{\partial y_i^j}{\partial t_f} \leq 0. \tag{24}$$

$$\frac{\partial y_i^j}{\partial \beta} < 0. \quad \frac{\partial y_i^j}{\partial v} \leq 0. \tag{25}$$

$$\frac{\partial y_i^j}{\partial e} \geq 0, j \in \{AT, IT\}. \quad \frac{\partial y_i^{IP}}{\partial e} < 0. \tag{26}$$

To understand the intuition behind these comparative statics results, we rewrite the MNE's first-order conditions in terms of its own outputs for the domestic and foreign markets (that is, we substitute in the reaction functions of its domestic and foreign competitors) as follows:

$$(1 - t_d)\frac{n + a - b(n+2)y_d^j}{n+1} = (1 - t_d)\beta + (1 - t_f)v y_f^j, \tag{27}$$

$$e\frac{h + m - k(m+2)y_f^j}{m+1} = \beta\varepsilon + v y_d^j, \tag{28}$$

for $j \in \{IT, IP\}$. The marginal revenue is on the left-hand side and the marginal cost is on the right-hand side. Notice that, while marginal revenue functions are independent across markets, marginal cost functions are not for the MNE ($v < 0$).

The comparative statics results in Lemma 4 are not surprising, with the exception of those with respect to the tax rates, t_d and t_f. As expected, the MNE's domestic and foreign output increases with market size, decreases with the elasticity of demand, and increases with competition in the two markets. An increase in the MNE's efficiency (that is, lower β) increases output levels in the two markets. The output levels are also increasing in the degree of complementarity, v; that is, the lower the v (the more negative), the higher the output level, y_i^j.

The more surprising result is that domestic and foreign taxes have opposite effects on the MNE's output levels in both markets. An increase in the domestic tax rate, t_d, reduces both the marginal revenue on the left-hand side of (27) and the marginal cost on the right-hand side of (27). The MNE's first-order condition for its foreign market output is unaffected by a change in t_d. The net effect of the reduction in marginal revenue and marginal cost of domestic

output production is to increase domestic output, because the effect on marginal cost, given by β, is greater than the effect on marginal revenue, $(n+a-b(n+2)y_d^j)/(n+1) = (1-\eta_{P_d,y_d})P_d$. To see why this is so, consider a restatement of (27): $(1-t_d)[(1-\eta_{P_d,y_d})P_d - \beta] - (1-t_f)\upsilon = 0$. Since the second term on the left-hand side is positive (due to $\upsilon < 0$), the expression $(1-\eta_{P_d,y_d})P_d - \beta$ must be negative. The increase in domestic output reduces the marginal cost of producing foreign output due to complementarities in production; hence, foreign output also increases.

On the other hand, an increase in the foreign tax rate, t_f, increases the marginal cost of domestic output by $-\upsilon y_f^j$. The MNE responds to this by decreasing domestic output. This also leads to a decrease in foreign output. An increase in the foreign profit tax, t_f, thus results in a decrease of output in both markets.

2.2 Comparison across cases

Since the purpose of this paper is to examine how the presence of an MNE affects the exchange rate pass-through relative to AT, and how the MNE's location of production matters for exchange rate pass-through, we need to compare equilibria across the cases of AT when an MNE is absent, and IT and IP when an MNE is present.

Proposition 4 *Domestic and foreign output by an MNE is higher compared with AT, assuming that ϵ is not too large:*

$$y_d^{IT},\ y_d^{IP} > y_d^{AT} \tag{29}$$

$$y_f^{IT} > y_f^{AT} \tag{30}$$

$$y_i^{IP} > y_f^{AT}, \quad \epsilon \leq 1 \text{ and } \epsilon > 1 \text{ but not too large.} \tag{31}$$

If $\epsilon < 1$, both the MNE's domestic and foreign outputs increase with a shift from IT to IP; that is, $y_i^{IP} \geq y_i^{IT}$, $i = d, f$. If $\epsilon > 1$, the reverse is true; that is, $y_i^{IP} \leq y_i^{IT}$.

Proof: This can be easily shown by directly comparing y_i^j, for $i \in \{d, f\}$, $j \in \{AT, IT, IP\}$. *Q.E.D.*

The intuition is as follows. When $\epsilon < 1$, the average marginal cost of the MNE falls with IP and, therefore, output increases. This increase is true for both the domestic and foreign markets due to the economies of scope the MNE enjoys. The opposite argument holds if

$e > 1$.

Proposition 5 *In response to an increase in the exchange rate, e, the MNE's outputs increase under IT while they decrease under IP*

$$E^{IT}_{y_i,e} < 0 < E^{IP}_{y_i,e}, \ i \in \{d, f\}. \tag{32}$$

Since $\eta_{P_i y_i} > 0$, $i \in \{d, f\}$, market prices fall in response to an increase in the exchange rate, $E^{IT}_{P_i,e} < 0$ ($i \in \{d, f\}$), for an MNE with IT. On the other hand, under an MNE with international production, market prices increase in response to an increase in the exchange rate, $E^{IP}_{P_i,e} > 0$, for $i \in \{d, f\}$.

Proof: The proof is relegated to Appendix F. *Q.E.D.*

The next proposition compares exchange rate pass-through under the different cases of an arm's-length exporter, and MNE with IT and an MNE with IP.

Proposition 6 *The exchange rate pass-through into domestic and foreign prices is incomplete under AT or an MNE; that is, $|E^j_{P_i,e}| < 1$, for $i \in \{d, f\}$ and $j \in \{AT, IT, IP\}$. Moreover,*

(i) introducing an MNE increases the exchange rate pass-through into domestic prices and reduces the exchange rate pass-through into foreign prices, regardless of the MNE's location of production:

$$0 = E^{AT}_{P_d,e} < |E^j_{P_d,e}|, \quad |E^{AT}_{P_f,e}| > |E^j_{P_f,e}|, \tag{33}$$

for $v < 0$ but close to zero, where $j \in \{IT, IP\}$ corresponds to the case with an MNE;

(ii) the exchange rate pass-through is higher under IT than under IP:

$$|E^{IT}_{P_i,e}| > E^{IP}_{P_i,e}, \tag{34}$$

for $i \in \{d, f\}$, $v < 0$ but close to zero and $e > \beta/(m+2)$.

Proof: The proof is relegated to Appendix G. *Q.E.D.*

Table 1: Comparison across cases

$\epsilon < 1$	$\epsilon > 1$
$y_d^{AT} < y_d^{IT} < y_d^{IP}$ $y_f^{AT} < y_f^{IT} < y_f^{IP}$	$y_d^{AT} < y_d^{IP} < y_d^{IT}$ $y_f^{AT} < y_f^{IP} < y_f^{IT}$
$\eta^{IT}_{P_d,y_d} \leq \eta^{IP}_{P_d,y_d}$ $\eta^{IT}_{P_f,y_f} \leq \eta^{IP}_{P_f,y_f}$	$\eta^{IT}_{P_d,y_d} \geq \eta^{IP}_{P_d,y_d}$ $\eta^{IT}_{P_f,y_f} \geq \eta^{IP}_{P_f,y_f}$
$E^{IT}_{y_d,e} < 0 < E^{IP}_{y_d,e}$ $E^{IT}_{y_f,e} < 0 < E^{IP}_{y_f,e}$	
$0 = E^{AT}_{P_d,e} < E^{IP}_{P_d,e} < \lvert E^{IT}_{P_d,e}\rvert < 1$ $0 < E^{IP}_{P_f,e} < \lvert E^{IT}_{P_f,e}\rvert < \lvert E^{AT}_{P_f,e}\rvert < 1$	

Table 1 summarizes the results in Propositions 4–6.

Proposition 6 shows that exchange rate pass-through is incomplete, $|E^{AT}_{P_f,e}| < 1$, due to imperfect competition, a common result in the standard industrial organization models of exchange rate pass-through.

Proposition 6 implies that introducing an MNE (i) increases the exchange rate pass-through into domestic prices and decreases the exchange rate pass-through into foreign prices, and (ii) always causes pass-through to be higher under IT than under IP. The intuition is as follows. An AT exporter's domestic output is invariant to exchange rate changes and, therefore, the exchange rate pass-through into domestic prices is zero under AT.[8] This is obvious from the first-order condition (27) for $\psi = 0$. An MNE's domestic output is, however, affected by exchange rate changes, because of linkages between the domestic and foreign markets that arise from economies of scope; that is, $\psi < 0$. The exchange rate pass-through into domestic prices is therefore positive in the presence of an MNE and, consequently, greater than under AT. On the other hand, exchange rate pass-through into foreign prices is lower in

[8]This is an artifact of the assumption that there are no exporters based in the foreign country. Modifying our model to allow for competition by other exporters will lead to a non-zero exchange rate pass-through under AT, perhaps a more realistic scenario.

the presence of an MNE than under AT, since the MNE is more "diversified" compared with the AT exporter. An MNE adjusts both foreign and domestic production in response to an exchange rate change due to linkages between the two markets that result from economies of scope. We can easily see this from the first-order conditions (27) and (28). A change in the exchange rate, ϵ, affects the marginal revenue of producing for the foreign market, and the MNE will adjust foreign output accordingly. This, in turn, affects the marginal cost of domestic production (that is, the left-hand side of (27)), and the MNE adjusts domestic output as well. This is no longer the case for an arm's-length exporter that adjusts only its foreign output in response to a change in the exchange rate. Since the MNE has two degrees of freedom, it does not have to adjust foreign output as much as the AT exporter. The output adjustment is then passed through into prices via changes in the elasticity of market demands with respect to output. This, in turn, renders foreign prices less sensitive to exchange rate movements in the presence of an MNE compared with AT.

For part (ii) of Proposition 6, the intuition is as follows. Under IP, a change in the exchange rate makes production in one market more expensive, and makes it less expensive in the other market. These two effects are offsetting. Under IT, however, a change in the exchange rate makes domestic production more or less expensive without an offsetting effect in the foreign market. As a consequence, the MNE must adjust domestic output by a greater extent under IT than under IP. The MNE is therefore more "diversified" under IP than under IT, and output is less sensitive to exchange rate movements under IP than under IT. The output adjustment translates into price adjustment via changes in the elasticity of market demands with respect to output. As a result, IT prices are more sensitive to exchange rate movement than their IP counterparts. Analytically, we can see this from the first-order conditions (27) and (28). Under IP, which corresponds to $v' < 0$ and $\varepsilon = \epsilon$, a change in the exchange rate, ϵ, affects both marginal revenue and marginal cost of foreign production in (28), since we use the domestic currency as numeraire. Under IT, which corresponds to $v' < 0$ and $\varepsilon = 1$, a change in the exchange rate affects only the marginal revenue of foreign production in (28). The MNE must adjust foreign production more under IT than under IP because there is no offsetting marginal cost effect.

To complete our analysis of how the presence of an MNE affects exchange rate pass-through, the following three propositions examine how exchange rate pass-through is affected by taxes, competition, and exchange rate movements. The results are obtained by taking partial derivatives of pass-through elasticities and directly inspecting these derivatives.

Proposition 7 *Domestic taxes increase, and foreign taxes reduce, the exchange rate pass-through in both domestic and foreign markets. Neither domestic nor foreign taxes affect the exchange rate pass-through in both domestic and foreign markets under AT:*

$$\frac{\partial |E^j_{P_i,e}|}{\partial t_d} > 0, \quad i \in \{d, f\}, \ j \in \{IT, IP\}. \tag{35}$$

$$\frac{\partial |E^j_{P_i,e}|}{\partial t_f} < 0, \quad i \in \{d, f\}, \ j \in \{IT, IP\}. \tag{36}$$

$$\frac{\partial |E^{AT}_{P_i,e}|}{\partial t_k} = 0, \quad i, k \in \{d, f\}. \tag{37}$$

Proposition 7 shows that domestic and foreign taxes have asymmetric effects on exchange rate pass-through. This result emphasizes the importance of distinguishing between the domestic and foreign markets when analyzing exchange rate pass-through.

Proposition 8 *When the MNE's domestic market becomes more competitive, exchange rate pass-through falls for all cases but one, the pass-through to foreign prices under IP:*

$$\frac{\partial |E^j_{P_d,e}|}{\partial n} < 0, \ j \in \{IT, IP\}. \tag{38}$$

$$\frac{\partial |E^{AT}_{P_f,e}|}{\partial n} = 0, \quad \frac{\partial |E^{IT}_{P_f,e}|}{\partial n} > 0, \quad \frac{\partial |E^{IP}_{P_f,e}|}{\partial n} < 0. \tag{39}$$

On the other hand, when the MNE's foreign market becomes more competitive, exchange rate pass-through increases for domestic prices and falls for foreign prices:

$$\frac{\partial |E^j_{P_d,e}|}{\partial m} > 0, \ j \in \{IT, IP\}. \tag{40}$$

$$\frac{\partial |E^j_{P_f,e}|}{\partial m} < 0, j \in \{AT, IT, IP\}. \tag{41}$$

Our results regarding the effects of competition on exchange rate pass-through when an MNE is present contrast with the results reported in the pricing-to-market literature on exchange rate pass-through that features AT only and competition with other exporters in the domestic market. In that literature, an increase in competition in the domestic market always increases the sensitivity of domestic prices to exchange rate movements. In our context, the price

sensitivity of a given market increases only in some cases, and, in those cases, always in response to an increase in the competition the MNE faces in the other market.

Proposition 9 *An appreciation/depreciation of the domestic currency leads to a decrease/increase in pass-through into both domestic and foreign prices:*

$$\frac{\partial |E^j_{P_i,e}|}{\partial e} \leq 0, \ i \in \{d, f\}, \ j \in \{AT, IT, IP\}. \tag{42}$$

3. Conclusions

In this paper, we develop a model that allows us to look at the effects of MNEs' pricing behaviour on the sensitivity of prices to exchange rate movements. Our simple model allows us to draw some powerful conclusions.

We first find that the exchange rate pass-through into domestic and foreign prices is incomplete. We also show that IT domestic prices are more sensitive to exchange rate movements than AT prices, whereas IT foreign prices are less sensitive to exchange rate movements than AT prices. Moreover, IT domestic and foreign prices are more sensitive to exchange rate movements than their IP counterparts.

Our results are consistent with some of the empirical evidence on exchange rate pass-through. First, the empirical evidence is somewhat mixed with respect to the sensitivity of IT prices to exchange rate movements compared with that of AT prices. Our results imply that it is important to distinguish between the domestic market (that is, the location of the MNE's parent) and the foreign market (the location of the subsidiary) when looking at the sensitivity of IT prices versus AT and IP prices. This could potentially explain why some empirical studies find IT prices more sensitive to exchange rate movements and others find them less sensitive.

Second, the empirical evidence shows that the exchange rate pass-through into U.S. import prices is lower than into Canadian import prices. One explanation in our model could be that foreign MNEs choose to deliver goods into the United States mainly by IP and into Canada mainly by IT. This is, of course, something that would have to be tested empirically.

Our analysis also shows that exchange rate pass-through does not always increase with competition, which is somewhat in contradiction with the pricing-to-market literature. The

pricing-to-market literature, however, considers only arm's-length exporters that produce at constant marginal cost. This is not the case in our model of the MNE, which could explain our mixed results.

In this paper, we abstract from intermediate production and assume that the MNE produces a homogeneous final good for two locations. In order to better compare our results with those in the standard industrial organization models of exchange rate pass-through, we need to consider intermediate production in our future work. This is important, since an increasing proportion of MNEs' trade is accounted for by intermediate goods. Relaxing the homogeneity assumption and introducing product differentiation would also allow us to look explicitly at transfer prices. In our model, the MNE competes only with purely domestic and foreign firms; we do not consider competition from exporters. Extending the analysis to include competition from exporters is left for future research.

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Appendix A: Proof of Lemma 1

Using $P_d = a - bY_d$, $P_f = h - kY_f$, $Y_d = y_d + ny_{-d}$, and $Y_f = y_f + my_{-f}$ in the first-order conditions (9) and (10), we obtain:

$$by_d + b(n+1)y_{-d} = a - 1. \tag{A.1}$$

$$ky_f + k(m+1)y_{-f} = h - 1. \tag{A.2}$$

Equations (A.1) and (A.2) can be solved for y_{-d} and y_{-f} in terms of y_d and y_f, respectively. The solutions give (11). Substituting (11) into the domestic and foreign aggregate output gives (12).

Appendix B: Proof of Proposition 1

The proof is straightforward:

$$\begin{aligned} E_{P_i,c} &= \frac{\partial P_i}{\partial c}\frac{c}{P_i} \\ &= \left[-\frac{\partial P_i}{\partial y_i}\frac{y_i}{P_i}\right]\cdot\left[-\frac{\partial y_i}{\partial c}\frac{c}{y_i}\right] \\ &= E_{y_i,c}\cdot\eta_{P_i,y_i}. \end{aligned} \tag{B.1}$$

Appendix C: Proof of Lemma 2

The price elasticities with respect to the MNE's outputs are, by definition:

$$\eta_{P_i,y_i} = -\frac{\partial P_i}{\partial y_i}\frac{y_i}{P_i}. \tag{C.1}$$

Since $P_d = -b$ and $P_f = -k$, we can use (12) to obtain the result shown in Lemma 2.

Appendix D: Proof of Lemma 3

The first-order condition with respect to the MNE's domestic output, (7), can be rewritten as:

$$\begin{aligned} \frac{a+n-b(n+2)}{n+1} &= SAC_{y_d} + \frac{1-t_f}{1-t_d} IC_{y_d} \\ &= \beta + \frac{1-t_f}{1-t_d} c y_f. \end{aligned} \tag{D.1}$$

Differentiating both sides of (D.1) with respect to e and multiplying by e/y_d gives:

$$\frac{b(n+2)}{n+1}\left(-\frac{\partial y_d}{\partial e}\right)\frac{e}{y_d} = \frac{1-t_f}{1-t_d} c \frac{\partial y_f}{\partial e}\frac{e}{y_f}\frac{y_f}{y_d}. \tag{D.2}$$

which can also be written as:

$$E_{y_d,e} = -c\frac{1-t_f}{1-t_d}\frac{n+1}{b(n+2)}\frac{y_f}{y_d}E_{y_f,e}. \tag{D.3}$$

which proves (19). The proof for (20) is similar. The first-order condition with respect to the MNE's foreign output, (8), can be rewritten as:

$$e\frac{h+m-k(m+2)y_f}{m+1} = \beta z + c y_d. \tag{D.4}$$

Differentiating both sides of (D.4) with respect to y_f, multiplying both sides by e/y_f, and rearranging terms gives (20).

Appendix E: Equilibrium Quantities and Prices

Solving equations (7)–(10) y_d, y_{-d}, y_f, and y_{-f} using the expressions in Lemma 1, we obtain:

(i) equilibrium quantities and prices in the domestic market,

$$y_d^j = \frac{1}{D}\{(1-t_d)ek(m+2)[a+n-\beta(n+1)] - (1-t_f)c\cdot(n+1)[e(h+m)-\varepsilon\beta(m+1)]\}, \quad \text{(E.1)}$$

$$y_{-d}^j = \frac{1}{D}\Big\{(1-t_d)ek(m+2)(a-2+\beta)+(1-t_f)c\cdot[e(h+m)-\beta(m+1)]-(1-t_f)c^2(m+1)\frac{a-1}{b}\Big\}, \quad \text{(E.2)}$$

$$Y_d^j = \frac{1}{D}\Big\{(1-t_d)ek(m+2)[a(n+1)-n-\beta]-(1-t_f)c\cdot[e(h+m)-\varepsilon\beta(m+1)]-(1-t_f)c^2n(m+1)\frac{a-1}{b}\Big\}, \quad \text{(E.3)}$$

$$P_d^j = \frac{1}{D}\Big\{(1-t_d)ekb(m+2)[a+n+\beta]+(1-t_f)c\cdot b[e(h+m)-\varepsilon\beta(m+1)]-(1-t_f)c^2(m+1)(a+n)\Big\}, \quad \text{(E.4)}$$

(ii) equilibrium quantities and prices in the foreign market,

$$y_f^j = \frac{1}{D}\{(1-t_d)b(n+2)[e(h+m)-\varepsilon\beta(m+1)] - (1-t_d)c\cdot(m+1)[a+n-\beta(n+1)]\}, \quad \text{(E.5)}$$

$$y_{-f}^j = \frac{1}{D}\Big\{(1-t_d)b(n+2)[e(h-2)+\varepsilon\beta]+(1-t_d)c\cdot[a+n-\beta(n+1)]-(1-t_f)c^2(m+1)\frac{h-1}{k}\Big\}, \quad \text{(E.6)}$$

$$Y_f^j = \frac{1}{D}\Big\{(1-t_d)cb(n+2)\{e[(m+1)h-m]-\varepsilon\beta\}-(1-t_d)c\cdot[a+n-\beta(n+1)]-(1-t_f)c^2m(n+1)\frac{h-1}{k}\Big\}, \quad \text{(E.7)}$$

$$P_f^j = \frac{1}{D}\Big\{(1-t_d)kb(n+2)[e(h+m)+\varepsilon\beta]+(1-t_d)c\cdot k[a+n-\beta(n+1)]-(1-t_f)c^2(n+1)(h+m)\Big\}, \quad \text{(E.8)}$$

for $j \in \{IT, IP\}$, where $D \equiv (1-t_d)ekb(n+2)(m+2) - (1-t_f)c^2(n+1)(m+1) > 0$.

Appendix F: Proof of Proposition 5

We can easily show that $E^{IT}_{y_d,e} < E^{IP}_{y_d,e}$:

$$\begin{aligned} E^{IT}_{y_d,e} &= \frac{1}{D}(1-t_f)c^2(n+1)(m+1) + \frac{1}{Dy^{IT}_d}(1-t_f)c\beta(n+1)(m+1) \\ &= E^{IP}_{y_d,e} + \frac{1}{Dy^{IT}_d}(1-t_f)c\beta(n+1)(m+1) \\ &< E^{IP}_{y_d,e}. \end{aligned} \tag{F.1}$$

We can easily derive the elasticities of the foreign outputs with respect to the exchange rate under IT and IP, respectively, and show that they are:

$$E^{IT}_{y_f,e} = -\frac{e\Delta^{IT}}{Dy^{IT}_f} < 0, \tag{F.2}$$

$$E^{IP}_{y_f,e} = -\frac{e\Delta^{IP}}{Dy^{IP}_f} > 0, \tag{F.3}$$

where

$$\Delta^{IT} = D(1-t_d)(n+2)b(h+m) - Dy^{IT}_f(1-t_d)(n+2)(m+2)kb > 0, \tag{F.4}$$

$$\Delta^{IP} = D(1-t_d)(n+2)b[(h+m) - (m+1)\beta] - Dy^{IP}_f(1-t_d)(n+2)(m+2)kb < 0, \tag{F.5}$$

for c close to zero. This implies that $E^{IT}_{y_f,e} < E^{IP}_{y_f,e}$ for c close to zero.

Using the results in Proposition 4, we can also show that $|\Delta^{IP}| > \Delta^{IT}$, which implies that $|E^{IT}_{y_f,e}| < E^{IP}_{y_f,e} < 1$.

Appendix G: Proof of Proposition 6

(i) The exchange rate pass-through into domestic and foreign prices is:

$$E^{IT}_{P_d,e} = \frac{1}{De}\{(1-t_f)v^2(m+1)[a+n-(n+1)P_d^{IT}] + (1-t_f)vb\beta(m+1)\}. \tag{G.1}$$

$$E^{IP}_{P_d,e} = \frac{1}{De}(1-t_f)v^2(m+1)[a+n-(n+1)P_d^{IP}]. \tag{G.2}$$

Since, $E^{IT}_{P_d,e} < 0$ and $E^{IP}_{P_d,e} > 0$, we compute the difference between the absolute value of $E^{IT}_{P_d,e}$ and $E^{IP}_{P_d,e}$ in order to compare the sensitivity of the domestic prices to exchange rate movements under IT and IP:

$$|E^{IT}_{P_d,e}| - E^{IP}_{P_d,e} = -\frac{1}{De}(1-t_f)v(m+1)[b\beta + 2v(a+n) - v(n+1)(P_d^{IT} - P_d^{IP})]. \tag{G.3}$$

For v close to zero, the left-hand side of (G.3) is positive and, therefore, $|E^{IT}_{P_d,e}| > E^{IP}_{P_d,e} > 0 = E^{AT}_{P_d,e}$.

In order to show that the exchange rate pass-through into domestic prices is incomplete under both IT and IP, it is enough to show that $|E^{IT}_{P_d,e}| < 1$. For v close to zero:

$$\begin{aligned}|E^{IT}_{P_d,e}| - 1 = &\frac{1}{De}\{(1-t_f)v^2(m+1)[a+n-(n+1)P_d^{IT}] - (1-t_f)b\beta(m+1)\\ &- (1-t_d)v^2kb(n+2)(m+2) + (1-t_f)ev^2(n+1)(m+1)\} < 0.\end{aligned} \tag{G.4}$$

(ii) The exchange rate pass-through into foreign prices under AT, IT, and IP, respectively, is:

$$E^{AT}_{P_f,e} = -\frac{1}{De}(1-t_d)\beta kb(n+2) < 0. \tag{G.5}$$

$$E^{IT}_{P_f,e} = \frac{1}{De}\{(1-t_f)v^2(n+1)[h+m-(m+1)P_f^{IT}] - (1-t_d)k\{v[a+n-\beta(n+1)] + b\beta(n+2)\}\} < 0, \tag{G.6}$$

$$E^{IP}_{P_f,e} = \frac{1}{De}\{(1-t_f)v^2(n+1)[h+m-(m+1)P_f^{IP}] - (1-t_d)vk[a+n-\beta(n+1)]\} > 0. \tag{G.7}$$

for v close to zero. We can show that foreign prices are more sensitive to exchange rate movements under AT than under IT by calculating the difference, $|E^{AT}_{P_f,e}| - |E^{IT}_{P_f,e}|$, and show that it is positive:

$$\begin{aligned}|E^{AT}_{P_f,e}| - |E^{IT}_{P_f,e}| = &-\frac{1}{De}v\{(1-t_f)v(n+1)[h+m-(m+1)P_f^{IT}]\\ &- (1-t_d)k[a+n-\beta(n+1)] - \frac{1}{DD^{AT}}v(n+1)(m+1)\} > 0,\end{aligned} \tag{G.8}$$

for v close to zero, where $D^{AT} = (1-t_d)ekb(n+2)(m+2)$. Also,

$$\begin{aligned}|E^{IT}_{P_f,e}| - E^{IP}_{P_f,e} = &\frac{1}{De}\{(1-t_f)v^2(n+1)(m+1)(P_f^{IT} - P_f^{IP}) - 2(1-t_f)v^2(n+1)(h+m)\\ &+ 2(1-t_d)kv[a+n-\beta(n+1)] + (1-t_d)bk\beta(n+2)\} > 0,\end{aligned} \tag{G.9}$$

for v close to zero.

So far, we have shown that $E^{IP}_{P_f,\epsilon} < |E^{IT}_{P_f,\epsilon}| < |E^{AT}_{P_f,\epsilon}|$ for ϵ sufficiently close to zero. Since

$$|E^{AT}_{P_f,\epsilon}| - 1 = \frac{1}{D\epsilon}(1 - t_d)kb(n+2)[\beta - \epsilon(m+2)] < 0, \tag{G.10}$$

for $\epsilon > \beta/(m+2)$, it follows that the exchange rate pass-through into foreign prices is incomplete under AT, IT, and IP.

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